[11] "Alcoa hydral 700 series," Alcoa Chemical Co., Pittsburgh, Pa. (1962).
[12] Thornton, M. J., "The size analysis of phenothiazine," J. Pharm. Pharmcol. 11, 127T–138T (1959).
[13] Kaye S. M., Middlebrooks, D. E., and Weingarten, G., "Evaluation of the Sharples Micromerograph for particle size distribution analysis," Feltman Research Labs., Picatinny Arsenal TR FRL-TR-54 (February 1962).
[14] Roby, H., private communication, Aerojet-General Corp., Sacramento, Calif. (1963).
[15] Reynolds, J. E. and Clapper, T. W., "The manufacture of perchlorate part 2," Chem. Eng. Progr. 57, 94–97 (December 1961).
[16] Stairmand, C. J., in discussing "Methods of production and control" by P. V. Danckwerts, *Powders in Industry* (Society of Chemical Industry, London, 1961), p. 428.

Comments on "Some Effects of Planform Modification on the Skin Friction Drag"

Milton J. Thompson*
The University of Texas, Austin, Texas

In a recent Technical Note, Hopkins[1] has presented an analysis of the skin friction drag for a "cranked" wing of double trapezoidal planform. The procedure followed includes consideration of the variation in skin friction coefficient with Reynolds number because of changes in wing chord.

It is interesting to note that this same basic concept was employed thirty years ago by Upson and the present writer[2] in a study of the drag of single trapezoidal planforms. The earlier work began with an expression for the profile drag coefficient of a chordwise element of the wing in the form

$$C_{D_0} = h[RN]^n(a_1 + a_2t^2)(1 + a_3C_L^3) \tag{1}$$

in which t is the thickness ratio and C_L the section lift coefficient. Definitions of the coefficients h, a_1, a_2, and a_3 are obvious from Eq. (1).

For the case corresponding to Hopkins' analysis, we put $t = 0$ and $C_L = 0$ and obtain

$$C_{D_0} = h[RN]^n a_1 \tag{2}$$

with the Reynolds number now given in terms of the local chord y as

$$RN = V_0 y/\nu \tag{3}$$

The application of Eq. (2) to the profile drag calculation for the chordwise strip of the wing $y\Delta x$ gives

$$C_{D_0} = [RN]_m{}^n \phi a_1 \tag{4}$$

where the Reynolds number is based on the mean chord and

$$\phi = \frac{2^{n+1}h(1 - K_y{}^{n+2})}{(n + 2)(1 + K_y)^{n+1}(1 - K_y)} \tag{5}$$

Changing the notation so that $h = K$ and $K_y = 1/\lambda_2$, Eq. (5) becomes identical with that obtained by application of Hopkins' results in this restricted case.

In connection with Eq. (4) in the Hopkins paper, a slight error is to be noted in that the denominator of the term in the square brackets should read $(1 - \lambda_2)(\lambda_2)^{n+1}$ instead of $(1 - \lambda_2)(\lambda_2)^{n+2}$. This error apparently was not carried over into the computations leading to Figs. 2 and 3 of Ref. 1.

It may also be of interest to note that the earlier work of Ref. 2 included effects of section thickness ratio and lift coefficient as indicated by Eq. (1). Induced drag was also included in these considerations along with a combined treatment of both aerodynamic and structural factors.

Received October 26, 1964.
* Professor and Chairman, Department of Aerospace Engineering; also Associate Director, Defense Research Laboratory. Associate Fellow Member AIAA.

References

[1] Hopkins, E. J., "Some effects of planform modification on the skin friction drag," AIAA J. 2, 413–414 (1964).
[2] Upson, R. H. and Thompson, M. J., "The drag of tapered cantilevered airfoils," J. Aeronaut. Sci. 1, 168–177 (1934).

Comments on "Turbulent Mixing of Coaxial Jets"

George L. Mellor*
Princeton University, Princeton, N. J.

In the case of incompressible, fully developed, turbulent jet flows, Alpinieri[1] has stated that the conventional eddy viscosity assumption, $\epsilon = \kappa b|u_c - u_e|$ (where b is the jet half-width, u_e the constant external velocity, and u_c the centerline velocity), is only verified experimentally for the case $u_e = 0$, and therefore it is fair game to propose an entirely new assumption when $u_e > 0$. In an attempt to include the effects of variable density, he has apparently selected an assumption out of a set of possibilities all deemed equally probable but one that seems to fit his particular set of data.

However, in the far downstream region there is nothing to distinguish between wakes and jets (where $u_e > 0$, $u_c < u_e$) so that, for the constant density case, we find from the data of Cooper and Lutzky[2] and Carmody[3] that, for large x, $b \sim x^{1/3}$ and $u_e - u_c \sim x^{-2/3}$ which is in agreement with predictions based on the conventional eddy viscosity assumption[4] or on similarity considerations.[2] It follows from this that $\epsilon \sim x^{-1/3}$. Alpinieri's assumption, where in fact ϵ increases with x, is therefore in disagreement with this data.

Both Alpinieri and, in a related paper, Ferri, Libby, and Zakkay[5] object to the fact that, when $u_e = u_c$ and $\rho_e \neq \rho_c$, the conventional assumption leads to zero mixing of scalar quantities such as temperature or concentration. However, in the absence of upstream turbulence or velocity distortions this would, indeed, be evident to an observer travelling with the velocity $u = u_e = u_c$.

The answer seems to be that Refs. 2 and 3 cover a large range of x/d whereas Ref. 1 does not. Thus, Alpinieri is really dealing with the near downstream region with an assumption that, for the most part, involves variables appropriate to the far downstream region. The fact that mixing occurs when $u_e = u_c$ indicates that the turbulence is either present in the upstream flow or is subsequently generated by the velocity profile distortion due to the upstream boundary layers or both.

In the far downstream region where mixing is caused by turbulence generated by the jet or wake velocity profile, it seems evident that $\epsilon = \varkappa b|u_e - u_c|$ where, however, $\varkappa = \varkappa(\rho_c/\rho_e)$ and is still to be determined.†

Received October 22, 1964.
* Associate Professor, Department of Aerospace and Mechanical Sciences. Member AIAA.

† A possible clue might be contained in the analysis of the inviscid stability of a discontinuous jet or cylindrical vortex sheet; one finds that a small disturbance grows like $\exp(\sigma t)$ where $\sigma = [(\rho_e\rho_c f)^{1/2}/(\rho_e + \rho_c f)]k|u_c - u_e|$, k is the disturbance wave number, $f = I_0(kb)K_1(kb)/I_1(kb)K_0(kb)$ (Bessel function notation as used by Hildebrand), and b = jet diameter. The implication is that one might expect $\epsilon \sim \sigma b^2$ where $kb = O(1)$ and $f = O(1)$.

References

[1] Alpinieri, L. J., "Turbulent mixing of coaxial jets," AIAA J. 2, 1560–1567 (1964).

[2] Cooper, R. D. and Lutzky, M., "Exploratory investigation of the turbulent wakes behind bluff bodies," David Taylor Model Basin Rept. 963 (1955).

[3] Carmody, T., "Establishment of a wake behind a disc," American Society of Mechanical Engineers Paper 64-FE-3 (1964).

[4] Schlicting, H., *Boundary Layer Theory* (McGraw-Hill Book Co., Inc., New York, 1955).

[5] Ferri, A., Libby, P. A., and Zakkay, V., "Theoretical and experimental investigation of supersonic combustion," *Third Congress, International Council of the Aeronautical Sciences* (Spartan Books, Baltimore, Md., 1964); also Polytechnic Institute of Brooklyn Aeronautical Lab. PIBAL Rept. 712 (1962).

[6] Hildebrand, F. B., *Advanced Calculus for Engineers* (Prentice-Hall, Inc., Englewood Cliffs, N. J., 1949).

Reply by Author to G. L. Mellor

L. J. Alpinieri*

Aerospace Corporation, El Segundo, Calif.

MELLOR'S comment cites wake results that indicate definite power law variations of centerline velocity decay and mixing region widths that tend to agree with the conventional eddy viscosity assumption.

However, it should be noted that conclusions regarding such power law results should be made with caution. Recently, Murphy and Dickinson[1] have shown that the determination of the power law variation for the wake growth is susceptible to considerable error, particularly in what can best be termed the "near-downstream" region. Furthermore, Zakkay[2] has recently obtained a voluminous amount of data for coaxial jets which indicate that the centerline decay of concentration appears to best fit an x^{-2} variation which contradicts the results cited by Mellor.

Nevertheless, Mellor raises a valid question regarding the fact that ϵ increases with x, whereas in all incompressible cases, asymptotic forms have either tended towards zero or approached a constant value. The answer here may well lie in the fact that the experiments for which ϵ were derived were centered in the near-downstream region. As a result, the zone of validity may at first appear to be excessively restrictive. However, it should be noted that it is within this apparently restricted region that approximately 90% of the velocity and concentration equalization takes place, and therefore, this region would ordinarily be the one of maximum interest.

Finally, Mellor seeks to attribute the mixing at $U_e \cong U_c$ to upstream turbulence or velocity distortions due to upstream boundary layers. If such quantities were indeed the principal instigators of the mixing process, one would expect that a perturbation of the upstream boundary layers or initial profiles should produce a significant effect on the downstream concentration distributions. In order to investigate this point the wind tunnel described in Ref. 3 was used for the following simple test. A 1-in. length of 0.022-in. thick sandpaper roughness was attached to the outer surface, downstream end, of the central jet in order to perturb the boundary layer of the external stream. The height of the roughness was of the same order as the boundary-layer thickness of the outer jet. Nevertheless, a comparison of the downstream concentration profiles at $x/r = 15$, taken with and without roughness, indicated no measureable differences. This result suggests that mixing at $U_e \cong U_c$ is not principally dependent on the nature of the upstream boundary layers.

Received November 30, 1964.

* Member of the Technical Staff, Fluid Mechanics Department; formerly Research Associate, Polytechnic Institute of Brooklyn. Member AIAA.

References

[1] Murphy, C. H. and Dickinson, E. R., "Growth of the turbulent wake behind a supersonic sphere," AIAA J. 1, 339–349 (1963).

[2] Zakkay, V., Krause, E., and Wu, S., "Turbulent transport properties for axisymmetric heterogeneous mixing," AIAA Preprint 64-99 (January 1964).

[3] Alpinieri, L. J., "Turbulent mixing of coaxial jets," AIAA J. 2, 1560–1567 (1964).